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CONSEQUENCES OF HIGHER SYMMETRY FOR ELECTROMAGNETIC AND FOR WEAK TRANSITIONS

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In my lecture I shall review some work that has been done on the consequences of higher symmetry schemes for electromagnetic and for weak transitions. The higher symmetry schemes I shall consider will be those based on the simple compact Lie groups of rank two. The work on the subject that I shall review has been done by RUEGG [1], by COLEMAN and GLASHOW [2] and by CABIBBO and GATTO [3]. For the group theory concepts that will be used here we refer to SALAM's lectures [4] and to the article by BEHREND, DREITLIM, FRONSDAL and LEE [5].

1. GROUPS

We shall confine our attention to models based on the simple compact Lie groups of rank two. From the theory of Lie groups we know that there are four such groups (two of which are isomorphic to each other). They are:

- $SU(3)$, the special unitary group in 3 dimensions. It is called A_2 in Cartan's notations. It is the group of all unitary unimodular matrices in complex 3-dimensional space. The order of $SU(3)$ is 8.
- G_2 is one of the so-called exceptional groups. It is a subgroup of O_7 (for its characterization see reference [5], p. 26). The order of G_2 is 14.
- B_2 is the orthogonal group O_5 in five dimensions. Its order is 10.
- C_2 is isomorphic to B_2 . It is the group of unitary matrices in 4 dimensions that leave a non-singular antisymmetric matrix invariant. Its order is 10.

To define a model one has to decide on the assignment of the various particles (baryons and mesons) to particular representations of the group. Thus many different models can, in principle, be constructed for each of the above group, depending on the way one assigns the particles to the representations of the group.

We now review the simplest models one can construct.

2. MODELS

(a) $SU(3)$. We first consider the simplest models based on $SU(3)$, viz., the Sakata model and the Gell-Mann-Ne'eman model.

The Sakata Model - based on the following assignment

$$\begin{array}{ll} D^3(1,0): & p, n, \Lambda \\ D^8(1,1): & \text{mesons } (\pi, K, \chi). \end{array}$$

[The irreducible representations are, as usual, denoted by $D^n(a_1, a_2)$ where

n is the dimension of the representation and a_1, a_2 are non-negative integers such that the highest weight of the representation can be written as a linear combination with coefficients a_1 and a_2 of the two fundamental dominant weights of the group.] The weight diagram for the representation $D^3(1,0)$ is shown in Fig. 1.

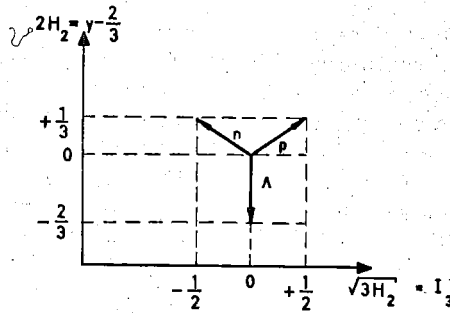


Fig. 1

Weight diagram for the representation $D^3(1,0)$

In this diagram H_1 and H_2 are the two commuting infinitesimal generators of the group. Each vector in the weight diagram is a simultaneous eigenvector of the commuting operators H_1 and H_2 with eigenvalues as indicated by the co-ordinates of its end-point. A convenient change of scale has been made by reporting on the co-ordinate axes the eigenvalues of $\sqrt{3} H_1$ and of $2H_2$ rather than those of H_1 and H_2 . In this way one sees that $\sqrt{3} H_1$ can be identified with I_3 (the third component of isotopic spin) and particles can be assigned to each weight vector in a definite way. One then checks that $2H_2$ can be related by hypercharge Y by the relation $2H_2 = Y - 2/3$. In this way one finds the relation, valid for the particular model, between the two conserved quantum numbers of the theory, I_3 and Y , and the two commuting operators H_1 and H_2 of the (rank two) group. Note that Y and I_3 are not, in the Sakata model that we are considering, simply multiples of H_1 and H_2 , but the relation between Y and H_2 is inhomogeneous. This circumstance is quite peculiar of the Sakata model and will not occur in the models we shall consider in the following. The relation between I_3, Y and H_1, H_2 , can only be homogeneous if Λ^0 and Σ^0 (which have $Y = I_3 = 0$) both lie at the centres of the weight diagrams for the representations to which they belong. In the Sakata model mesons belong to $D^8(1,1)$, whose weight diagram we shall discuss in connection with the eight-fold way. We also report the multiplication rules:

$$3 \times 3 = 1 + \boxed{8},$$

$$8 \times 8 = 1 + \boxed{8} + \boxed{8} + 10 + \overline{10} + 27.$$

We have put in a little square the regular representation. The eight-fold way of Gell-Mann and Ne'eman is based on the assignments

$D^8(1,1)$: baryons (N, Λ , Σ , Ξ),

$D^8(1,1)$: mesons (π , K, κ).

The weight diagram for $D^8(1,1)$ is shown in Fig. 2.

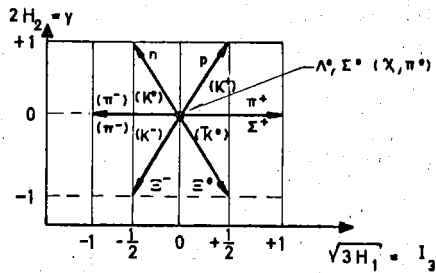


Fig. 2

Weight diagram for the representation $D^8(1,1)$

On the side of each vector we have reported the corresponding baryon and in brackets the corresponding meson. The multiplication rule is, of course, as before

$$8 \times 8 = 1 + \boxed{8} + \boxed{8} + 10 + \overline{10} + 27.$$

It is important to note that the regular representation 8 occurs twice in 8×8 .

(b). G_2 The model based on G_2 that we shall consider is based on the following assignment :

$D^2(0,0)$: Λ ,

$D^7(1,0)$: N, Σ , Ξ ,

$D^7(1,0)$: mesons (π , K).

The weight diagram for $D^7(1,0)$ is shown in Fig. 3.

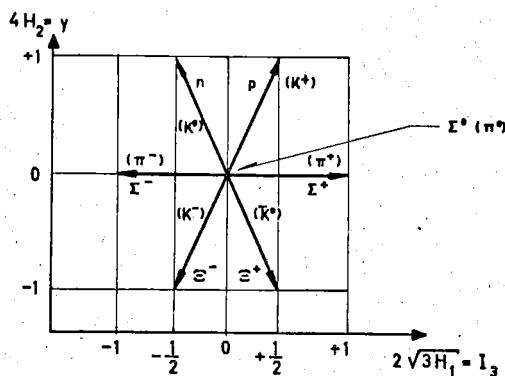


Fig. 3

Weight diagram for the representation $D^7(1,0)$

The multiplication rules are

$$\begin{aligned} 1 \times 1 &= 1, & 1 \times 7 &= 7, \\ 7 \times 7 &= 1 + 7 + \boxed{14} + 27. \end{aligned}$$

In the product 7×7 the regular representation $\boxed{14}$ occurs only once. It does not occur in the other products.

(c) B_2 . The model based on B_2 is the following:

$$\begin{aligned} D^1(0,0) &: \Lambda, \\ D^4(1,0) &: N \Xi, \\ D^5(0,1) &: \Sigma \times \text{(a new baryon)}, \\ D^4(1,0) &: K, \\ D^5(0,1) &: \pi \times \text{(a new meson)}. \end{aligned}$$

The weight diagram for $D^4(1,0)$ is shown in Fig. 4, and that for $D^5(0,1)$

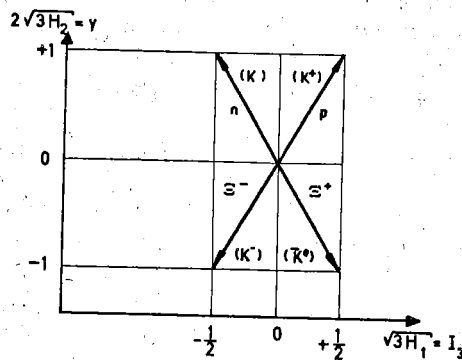


Fig. 4

Weight diagram for the representation $D^4(1,0)$

is shown in Fig. 5. The multiplication rules that one needs are

$$\begin{aligned} 1 \times 4 &= 4, \\ 1 \times 5 &= 5, \\ 4 \times 4 &= 1 + 5 + \boxed{10}, \\ 4 \times 5 &= 4 + 16, \\ 5 \times 5 &= 1 + \boxed{10} + 14. \end{aligned}$$

We see that the regular representation is only contained once in 4×4 and 5×5 .

(d) C_2 . We consider two models based on C_2 . We call the first model $[C_2]_I$ and the second $[C_2]_{II}$.

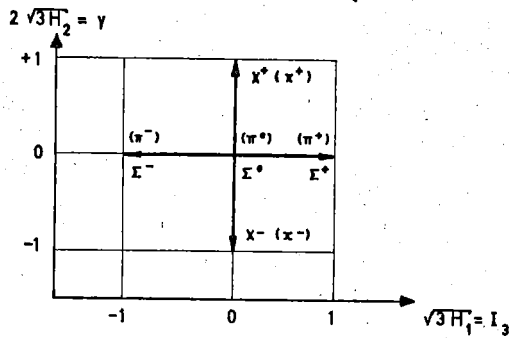


Fig. 5

Weight diagram for the representation $D^5(1,0)$

$[C_2]_I$ is based on the following assignment:

- $D^5(0,1) : \Lambda, N, \Xi,$
- $D^{10}(2,0) : \Sigma + \text{other baryons},$
- $D^{10}(2,0) : \pi, K, D \text{ (a new meson).}$

The weight diagram for $D^5(0,1)$ is shown in Fig. 6. The weight diagram for

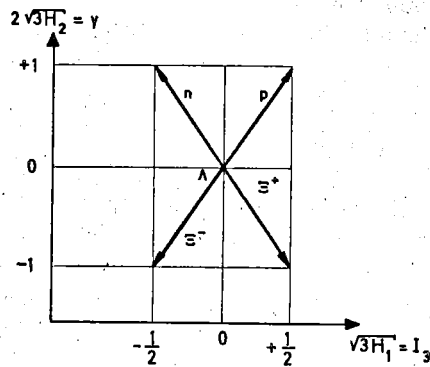


Fig. 6

Weight diagram for the representation $D^5(0,1)$

$D^{10}(2,0)$ is shown in Fig. 7. The relevant multiplication rules are

$$\begin{aligned}
 5 \times 5 &= 1 + \boxed{10} + 14, \\
 5 \times 10 &= 5 + \boxed{10} + 35, \\
 10 \times 10 &= 1 + 5 + 10 + 14 + 35 + 35'.
 \end{aligned}$$

Again the regular representation is only contained once.

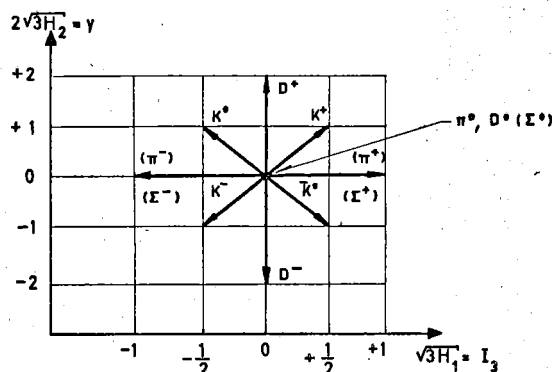


Fig. 7

Weight diagram for the representation $D^{10}(2,0)$

$[C_2]_{II}$ is instead based on the following assignment:

$$\begin{aligned} D^{10}(2,0) &: \text{ baryons,} \\ D^{(10)}(2,0) &: \text{ mesons.} \end{aligned}$$

The assignment of baryons to $D^{(10)}(2,0)$ is obvious and can be read from the weight diagram that we gave for $D^{10}(2,0)$. Two new baryons are required to fit the scheme. The relevant multiplication rule is 10×10 given above.

3. ELECTROMAGNETIC FORM FACTORS

The electromagnetic vertex $\langle A | j_\mu | A \rangle$ where A is a baryon or a meson and j_μ is the electromagnetic current operator which can be expressed in terms of the form factors of A :

$$\langle A | j_\mu | A \rangle = \bar{u}_A [F_1(k^2) \gamma_\mu \Omega + \dots] u_B$$

The matrix Ω in the above equation depends on the particular group-theoretical model.

For all the models that we have discussed, except for those based on $SU(3)$, it is very easy to derive the conditions that the group symmetry imposes on the form factors. In fact for the models that we called G_2 , B_2 , $[C_2]_I$ and $[C_2]_{II}$ the following circumstance holds: I_3 and Y are multiples of H_2 and H_1 . Therefore the charge $Q = I_3 + \frac{1}{2} Y$ is a linear homogeneous function of H_1 and H_2 . Therefore a realization of Ω is Q itself. But it is also the only possible realization since for G_2 , B_2 , $[C_2]_I$ and $[C_2]_{II}$ the regular representation is only contained once (at most) in the product $D^a \times D^a$ where D^a is one of the representations employed to describe the particles. It follows that all positively charged particles have the same form factor, all negatively charged particles the same form factor (equal and opposite to that of the

positively charged ones) and all neutral particles have form factors of zero. It is easy to generalize these remarks by including also transition matrix elements such as, for instance, that responsible for $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. Then in G_2 such a matrix element is zero, in B_2 it is also zero, while in $[C_2]_I$ it can be different from zero (and in fact it will in general be different from zero) and in $[C_2]_{II}$ is again zero by the general argument given before.

We next discuss the models based on $SU(3)$ which need a more detailed discussion. It is known from the general theory of Lie algebras that there exists a choice of the group generators F_m such that commutation relations are

$$[F_m, F_n] = i f_{mnl} F_l$$

with f_{mnl} real and completely antisymmetric. For $SU(3)$ the f_{mnl} are as follows [6]:

$$\begin{aligned} f_{123} &= 1, \\ f_{147} = f_{246} = f_{257} = f_{345} = f_{516} = f_{367} &= 1/2, \\ f_{458} = f_{678} &= \sqrt{3}/2. \end{aligned}$$

and the remaining ones can be obtained from the antisymmetry requirement. Since $f_{123} = 1$, one has the commutation relations

$$\begin{aligned} [F_1, F_2] &= i F_3, \\ [F_2, F_3] &= i F_1, \\ [F_3, F_1] &= i F_2, \end{aligned}$$

suggesting that F_1, F_2, F_3 are to be interpreted as I_1, I_2 and I_3 .

Next one looks for F_n that commutes with F_3 . From

$$[F_3, F_n] = i f_{3nl} F_l = 0$$

and from the values of the f_{mnl} it follows that only F_8 commutes with F_3 . Thus F_3 and F_8 are the two commuting elements of the Lie algebra. For the physical interpretation of F_8 one has to specify the model.

We first consider the eight-fold way. From the weight diagram for $D^8(1,1)$ of $SU(3)$ (see section 2) we learn that

$$\begin{aligned} \sqrt{3} H_1 &= F_3 = I_3, \\ 2 H_2 &= (2/\sqrt{3}) F_8 = Y. \end{aligned}$$

In analogy with

$$A = I_3 + \frac{1}{2} Y$$

the electromagnetic current is given by

$$j = j_3 + (1/\sqrt{3}) j_8,$$

where the currents j_m satisfy

$$[F_m, j_n(x)] = i f_{mnl} j_l(x),$$

i. e. they belong to the 8-dimensional (regular) representation. The group generator F_m are the space integrals of j_m :

$$F_m = \int d\sigma_\mu j_m^{(\mu)}(x).$$

Now it is easy to see that F_3 , F_8 , F_6 and F_7 commute with j . It is obvious that F_3 and F_8 commute with j , and the only physical implication of this fact is that j conserves both I_3 and Y . It can also easily be seen that

$$[F_6, j] = [F_6, j_3] + 1/\sqrt{3} [F_6, j_8] = 0,$$

using the values for f_{mnl} ; and similarly

$$[F_7, j] = 0.$$

The physical implications of the conservation of F_7 are the same as for the conservation of F_6 , so we shall only consider these last ones. From

$$[F_6, J(j)] = 0,$$

where $J(j)$ is any function of j , such as for instance a retarded product etc., we have

$$\langle A | [F_6, J(j)] | B \rangle = 0$$

or

$$\langle 0 | A [F_6, J(j)] B^+ | 0 \rangle = 0,$$

where A^* and B^* are the creation operators of the states $| A \rangle$ and $| B \rangle$.

Now for the vacuum

$$F_m | 0 \rangle = 0$$

since the vacuum state is assumed to be invariant under the group. So we can write the above equation as

$$\langle 0 | [A, F_6] J(j) | B \rangle = \langle A | J(j) [F_6, B^+] | 0 \rangle.$$

We specialize A and B to be one-particle states (baryons or mesons). In the eight-fold way both baryons and mesons belong to the regular eight-dimensional representation. By a suitable choice of the A 's one thus has

$$[F_m, A] = i f_{mAC} C.$$

Thus we find

$$f_{AbC} \langle C | J(j) | B \rangle = f_{BbC} \langle A | J(j) | C \rangle,$$

which constitute a set of identities to be satisfied by the matrix elements of $J(j)$ between one-particle states.

By specializing the above result one finds a number of consequences of which we list some:

For mesons:

The K^0 (or \bar{K}^0) form factor is identically zero;

The K^+ (K^-) form factor is identical to the π^+ (π^-) form factor;

The Compton effect matrix elements satisfy

$$\langle K^+ | j(x) j(x') | K^+ \rangle = \langle \pi^+ | j(x) j(x') | \pi^+ \rangle,$$

$$\sqrt{3} \langle K^0 | j(x) j(x') | K^0 \rangle = \langle \chi^0 | j(x) j(x') | \pi^0 \rangle - \sqrt{3} \langle \chi^0 | j(x) j(x') | \chi^0 \rangle,$$

$$- \langle K^0 | j(x) j(x') | K^0 \rangle = \langle \pi^0 | j(x) j(x') | \pi^0 \rangle - \sqrt{3} \langle \pi^0 | j(x) j(x') | \chi^0 \rangle.$$

For the 2γ decay modes:

$$(\text{amplitude for } \chi^0 \rightarrow 2\gamma) = (1/\sqrt{3}) (\text{amplitude for } \pi^0 \rightarrow 2\gamma).$$

This last equation will probably be useful in connection with the recent experiments that indicate a large branching ratio for $\chi^0 \rightarrow 2\gamma$.

For baryons:

$$\langle \Sigma^+ | j \dots j | \Sigma^+ \rangle = \langle p | j \dots j | p \rangle,$$

$$\langle \Sigma^- | j \dots j | \Sigma^- \rangle = \langle \Xi^- | j \dots j | \Xi^- \rangle,$$

$$\langle \Xi^0 | j \dots j | \Xi^0 \rangle = \langle n | j \dots j | n \rangle,$$

$$- 1/\sqrt{3} \langle \Sigma^0 | j \dots j | \Lambda \rangle = \langle n | j \dots j | n \rangle - \langle \Lambda | j \dots j | \Lambda \rangle,$$

$$- \sqrt{3} \langle \Lambda | j \dots j | \Sigma^0 \rangle = \langle n | j \dots j | n \rangle - \langle \Sigma^0 | j \dots j | \Sigma^0 \rangle,$$

where we have denoted briefly by $j \dots j$ a product $j(x) j(x') \dots j(x^{(n)})$.

From the last equations we find the relation between the electromagnetic mass splittings

$$\delta m_{\Xi^-} - \delta m_{\Xi^0} = \delta m_p - \delta m_n + \delta m_{\Sigma^-} - \delta m_{\Sigma^+}$$

(just by adding the first three equations).

The only information used up to this point has been the commutativity of the electromagnetic current $j(x)$ with F_8 . However we know more directly that

$$j(x) = j_3(x) + \frac{1}{\sqrt{3}} j_8(x)$$

so that if we have to compute a matrix element of the simple form

$\langle A | j(x) | B \rangle$ we can make use of the explicit form of j and express it in terms of a few reduced matrix elements. The procedure is quite analogous to the use of the Wigner-Eckart theorem, very common in problems involving the three-dimensional rotation group. Essentially since A and B belong to the 8-dimensional representation of $SU(3)$ and $j(x)$ also belongs to the regular representation, one has to extract from the direct product $8 \times 8 = 1 + 8 + 8 + 10 + 10 + 27$ the regular representation which is contained there twice. So one has two reduced matrix elements. The reduction formula is $\langle A | j_m | B \rangle = f_{ABm} \theta + d_{ABm} \zeta$, where f_{ABm} has already been reported, d_{ABm} is a completely symmetrical tensor (see later), and are reduced matrix elements (corresponding to the double occurrence of 8 in the product 8×8 , one time as 8-antisymmetrical and one time as 8 symmetrical). We shall sketch here an inelegant proof of this reduction formula by specializing to the three-dimensional representation of $SU(3)$ (the simplest non-trivial one). In such a representation one represents the elements of the Lie algebra F_m by matrices $\frac{1}{2} \lambda_m$, which satisfy

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k.$$

An explicit choice is the following:

$$\begin{aligned} \lambda_{1,2,3} &= \begin{bmatrix} \sigma_{1,2,3} & 0 \\ 0 & 0 \end{bmatrix} & \lambda_4 &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} & \lambda_5 &= \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \\ \lambda_6 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} & \lambda_7 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} & \lambda &= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \end{aligned}$$

The matrices λ are traceless and satisfy

$$\{\lambda_i, \lambda_j\} = 2 d_{ijk} \lambda_k + \frac{4}{3} \delta_{ij},$$

where d_{ijk} is a completely symmetric tensor with components

$$\begin{aligned} d_{118} = d_{228} = d_{338} = -d_{388} &= 1/\sqrt{3}, \\ d_{146} = d_{157} = -d_{247} = d_{256} = d_{344} = d_{355} = -d_{366} = -d_{377} &= 1/2, \\ d_{448} = d_{558} = d_{668} = d_{778} &= -i/2\sqrt{3}. \end{aligned}$$

The matrices are normalized such that

$$\text{Tr} (\lambda_i \lambda_j) = 2 \delta_{ij}.$$

Now, from the trilinear product $\lambda_A \lambda_m \lambda_B$ I can form two invariants, namely $\text{Tr} (\lambda_A \lambda_m \lambda_B)$ and $\text{Tr} (\lambda_B \lambda_m \lambda_A)$, or better, using the commutator and the anticommutator, $\text{Tr} ([\lambda_A, \lambda_B] \lambda_m)$ and $\text{Tr} (\{\lambda_A, \lambda_B\} \lambda_m)$. But

$$\text{Tr} ([\lambda_A, \lambda_B] \lambda_m) = 2i f_{ABK} \text{Tr} (\lambda_K \lambda_m) = 2i f_{ABK} \cdot 2 \delta_{Km} = 4i f_{ABm},$$

$$\text{Tr} (\lambda_A \lambda_B \lambda_m) = 2 \text{id}_{ABK} \text{Tr} (\lambda_K \lambda_m) + (4/3) \delta_{AB} \text{Tr} \lambda_m = 4 \text{id}_{ABm},$$

from which one gets the reduction formula. Applying the reduction formula to the electromagnetic current j one has

$$\langle A | j | B \rangle = i(f_{AB3} + (1/\sqrt{3}) f_{AB8}) \theta + (d_{AB3} + (1/\sqrt{3}) d_{AB8}) \xi.$$

In this way one finds:

For the mesons: the amplitudes for transitions

$$\text{vector meson} \rightarrow \text{pseudoscal. meson} + \gamma$$

are related by $(\rho^+ \rightarrow \pi^+ \gamma) = (K^{*+} \rightarrow K^+ \gamma)$,

$$(\rho^0 \rightarrow \eta \gamma) = (\rho^0 \rightarrow \pi^0 \gamma) = -(1/\sqrt{3}) (\omega^0 \rightarrow \eta \gamma) = (1/3) (\omega^0 \rightarrow \pi^0 \gamma) = -(2/\sqrt{3}) (K^{*0} \rightarrow K^0 \gamma).$$

For the baryons: one has the explicit expression of the form factors in terms of the two independent matrix elements θ and ξ

$$\begin{aligned} \langle \Sigma^0 | j | \Sigma^0 \rangle &= (1/3)\xi, & \langle \Xi^0 | j | \Xi^0 \rangle &= -(2/3)\xi, \\ \langle \Lambda^0 | j | \Lambda^0 \rangle &= -(1/3)\xi, & \langle p | j | p \rangle &= (1/3)\xi + \theta, \\ \langle \Sigma^0 | j | \Lambda^0 \rangle &= -(1/\sqrt{3})\xi, & \langle n | j | n \rangle &= -(2/3)\xi, \\ \langle \Sigma^- | j | \Sigma^- \rangle &= (1/3)\xi - \theta, & \langle \Sigma^+ | j | \Sigma^+ \rangle &= (1/3)\xi + \theta, \\ \langle \Xi^- | j | \Xi^- \rangle &= (1/3)\xi - \theta, \end{aligned}$$

We now discuss the electromagnetic form factors in the Sakata model. We have already derived the connection between I_3 and Y and the two commuting generators of SU_3

$$\begin{aligned} I_3 &= \sqrt{3} H_1, \\ Y &= (2/3) + 2 H_2. \end{aligned}$$

In terms of the generators F

$$\begin{aligned} I_3 &= F_3, \\ Y &= 2/3 + (2/\sqrt{3})F_8. \end{aligned}$$

Therefore the current is

$$j = (j_3 + (1/\sqrt{3})j_8) + (1/3)j_0$$

where the current j_0 is associated to the phase transformation that, added to $SU(3)$, produces $U(3)$. In reducing an electromagnetic vertex $\langle A | j | B \rangle$ for the three fundamental p, n, Λ , we use the multiplication rule $3 \times 3 = 1 + 8$.

So there is a contribution from 1 (corresponding to j_0) and a contribution from 8 (corresponding to j_3 and j_8). In the eight-fold way we had two reduced matrix elements because 8 was contained twice in 8×8 . Here we have again two reduced matrix elements but for a different reason (the appearance of a term j_0 in j). If we denote M_8 the matrix element for 8 and M_1 that for 1, we have

$$\langle A | j | B \rangle = (Q - 1/3) M_8 + (1/3) M_1$$

or explicitly

$$\langle p | j | p \rangle = (2/3) M_8 + (1/3) M_1,$$

$$\langle n | j | n \rangle = -(1/3) M_8 + (1/3) M_1,$$

$$\langle \Lambda | j | \Lambda \rangle = -(1/3) M_8 + (1/3) M_1.$$

In particular we expect for the Sakata model that n and X have the same anomalous moment. On the other hand the eight-fold way gives for the anomalous Λ magnetic moment one-half of that of n . Experiments are still uncertain to decide in favour of one of the two alternatives.

4. WEAK INTERACTIONS

One is tempted to assume that weak interactions are also expressed in terms of currents belonging to the regular representation. The $\Delta S = 0$ vector current is then given by $g(j_1 + i j_2)$ for $\Delta Q = +1$ and $g(j_1 - i j_2)$ for $\Delta Q = -1$; similarly for $\Delta S = +1$ one has $g'(j_4 + i j_5)$ for $\Delta Q = +1$ and for $\Delta S = -1$ $g'(j_4 - i j_5)$ for $\Delta Q = -1$. The weak constants g and g' are presumably not the same. If they were, one would expect, if string perturbations are to be excluded, a much faster rate for hyperon β -decay than that observed. With the same reasonings of the previous sections one derives easily

$$g \langle \Xi^- | j_4 + i j_5 | \Lambda \rangle = (1/\sqrt{2}) (\sqrt{3} \theta' - (1/\sqrt{3}) \zeta'),$$

$$g \langle \Sigma^- | j_4 + i j_5 | n \rangle = -\theta' + \zeta',$$

$$g \langle \Sigma^0 | j_4 + i j_5 | p \rangle = (1/\sqrt{2}) (-\theta' + \zeta'),$$

$$g \langle \Lambda | j_4 + i j_5 | p \rangle = (1/\sqrt{2}) (\sqrt{3} \theta' + \zeta'/\sqrt{3}),$$

$$g \langle \Xi^0 | j_4 + i j_5 | \Sigma^0 \rangle = (1/\sqrt{2}) (\theta' + \zeta'),$$

$$g \langle \Xi^0 | j_4 + i j_5 | \Sigma^+ \rangle = \theta' + \zeta'.$$

Unfortunately, currents with $\Delta S/\Delta Q = -1$ seem to be present and the point of view considered here of choosing the weak currents as belonging to the regular representation does not allow for currents with $\Delta S/\Delta Q = -1$. Currents with $\Delta S/\Delta Q = -1$ would require $\Delta T = 3/2$. One can try to relax the con-

dition that the weak currents belong to the regular representation. This might seem unpleasant in some respects (i. e. strangeness conserving vector part) but would allow some more freedom. So we shall look in the following for the simplest way in any of the models we are discussing to have currents with $T = 1, 1/2, 3/2$ and which could originate the observed decay modes. In $SU(3)$ the simplest (lowest dimensionality) representations containing $3/2$ are $D^{10}(3,0)$ and $\bar{D}^{10}(0,3)$, called briefly 10 and $\bar{10}$. We give the weight diagram of $D^{10}(3,0)$ in Fig. 8.

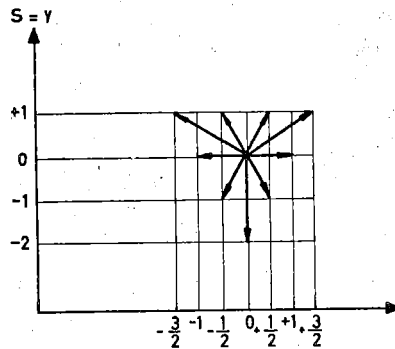


Fig. 8

Weight diagram for the representation $D^{10}(3,0)$

So one can obtain currents with $\Delta S = 1, \Delta T = 3/2, \Delta S = 0, \Delta T = 1, \Delta T = 1/2, \Delta T = 0, \Delta S = -2$. For symmetry reasons one would then have to introduce $\bar{10}$ also. Among the difficulties of such a scheme one is the presence of $\Delta S = \pm 2$ currents, which would lead for instance to $\Xi \rightarrow N + \ell + \nu$. All the amplitudes would be related in the eight-fold way, whereas in the Sakata model, since 3×3 does not contain 10 or $\bar{10}$, there would be no β -decay of baryons and the model would be inconvenient.

Passing now to G_2 , one needs currents belonging to $D^7(1,0)$ for $\Lambda \rightarrow N + e + \nu$ ($1 \times 7 = 7$) while for β -decay one has $7 \times 7 = 27 + 14 + 7 + 1$. However $D^7(1,0)$ does not contain $T = 3/2$ (isotopic content of $D^7(10)$ is $1/2, 1/2, 1$). The simplest representation containing $T = 3/2$ is $D^{14}(0,1)$. So one is lead to a superposition of D^7 and D^{14} . The isotopic content of D^{14} is $0, 0, 0, 1, 3/2$. Note that it does not contain $1/2$. If one wants $1/2$ and $3/2$ in the same representation, one has to use $D^{27}(2,0)$ with isotopic content $0, 1/2, 1/2, 1, 1, 1, 3/2, 3/2, 2$. But in this case one has to invert a particular treatment $\Lambda\beta$ -decay which only goes through D^7 . With a superposition of D^{14} and D^7 one has two reduced matrix elements ($7 \times 7 = 27 + 14 + 7 + 1$) for leptonic decays, with D^{27} only one reduced matrix elements.

Things do not get much more appealing with the other models. With B_2 it is typical that there are no representations that contain both integer and semi-integer spins. The simplest choice containing $T = 1$ is $D^5(0,1)$; the simplest containing $1/2$ and $3/2$ is $D^{16}(1,1)$. The isotopic contents are: for $D^5(0,1)$ $T = 0, 0, 1$; for $D^{16}(1,1)$ $T = 1/2, 1/2, 1/2, 1/2, 3/2, 3/2$. With such choice $\Lambda\beta$ decay ($1 \times 4 = 4$) would require a separate explanation. $\Sigma\beta$ -

decay and $N\beta$ -decay would be completely unrelated. One would need two reduced matrix elements and a separate explanation for $\Lambda\beta$ -decay.

In $[C_2]_I$ we need for decay some representation in the product $5 \times 5 = 14 + 10 + 1$, for $\Sigma\beta$ -decay some of $10 \times 5 = 35' + 10 + 5$, and for meson-decay some of $10 \times 10 = 35 + 35' + 14 + 10 + 5 + 1$. The regular representation 10 is contained in all of the products but does not contain $T = 3/2$ (its isotopic content is 0, 0, 0, 1/2, 1). The simplest to have a complete isotopic content is $D^{20}(3,0)$ (isotopic content 0, 0, 0, 0, 1/2, 1/2, 1/2, 1, 3/2), but it is not contained in any of the products so it is of no use. $D^{35}(2,1) \equiv 35'$ has a sufficient isotopic content but does not lead to nucleon β -decay. A possible way out would be a superposition of 10 and 35 or of 10 and 35' (the second choice (10 + 35') would allow $\Sigma\beta$ -decay with $\Delta T = 3/2$ while the first one would not).

Finally in $[C_2]_{II}$ we have only to choose one representation in the product $10 \times 10 = 35 + 35' + 14 + 10 + 5 + 1$ that has a complete isotopic content ($T = 1, 1/2$ and $3/2$). Both 35 and 35' can do it (35 has $T = 0, 0, 0, 0, 0, 1/2, 1/2, 1/2, 1/2, 1, 1, 1, 3/2, 3/2, 2$; 35' has $T = 0, 0, 0, 1/2, 1/2, 1/2, 1/2, 1/2, 1, 1, 1, 1, 3/2, 3/2$). By choosing 35' (no $T = 2$) one would have only one reduced matrix element for all the leptonic decays of baryons.

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